

Jamming transition in air transportation networks

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Abstract

In this work we present a model of an air transportation traffic system from the complex network modelling viewpoint. In the network, every node corresponds to a given airport, and two nodes are connected by means of flight routes. Each node is weighted according to its load capacity, and links are weighted according to the Euclidean distance that separates each pair of nodes. Local rules describing the behavior of individual nodes in terms of the surrounding flow have been also modelled, and a random network topology has been chosen in a baseline approach. Numerical simulations describing the diffusion of a given number of agents (aircraft) in this network show the onset of a jamming transition that distinguishes an efficient regime with null amount of airport queues and high diffusivity (free phase) and a regime where bottlenecks suddenly take place, leading to a poor aircraft diffusion (congested phase). Fluctuations are maximal around the congestion threshold, suggesting that the transition is critical. We then proceed by exploring the robustness of our results in neutral random topologies by embedding the model in heterogeneous networks. Specifically, we make use of the European air transportation network formed by 858 airports and 11170 flight routes connecting them, which we show to be scale-free. The jamming transition is also observed in this case. These results and methodologies may introduce relevant decision making procedures in order to optimize the air transportation traffic.

Key words: Complex systems, Complex networks, Air transportation

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1 Introduction

In the last decades Physics of Complex Systems and Complexity Science has started to address real world problems. In particular, much attention has been paid to self-driven particles such as pedestrian and freeway traffic [1,2] or Internet traffic systems [3]. Not that surprisingly, complex features such as the emergence of phase transitions [3,4,5,6,7] or criticality [8] take place in models that characterize these complex collective phenomena. Thereby, such complex systems focuses seem to be both a realistic and useful approach when describing the concept of traffic dynamics [1], both in homogeneous and in heterogeneous media.

The first insights considering traffic dynamics from a cooperative phenomenon point of view were developed in cellular automata. In [9] and subsequent works, Nagel and Shreckenberg developed a stochastic discrete model of freeway traffic dynamics which evidenced a free-congested phase transition quite similar to the real traffic behaviors. Successive refinements and generalizations of this model such as [4] or [5,7] have been performed so far. All these models focus on how the aggregation of local dynamical rules may generate emergent non-linear behaviors at the global level, such as travelling jam waves, for instance. Pedestrian dynamics has also been addressed from a complex system point of view. Self-organization effects occurring in pedestrian crowds which lead to unexpected mutual disturbances of pedestrian flows, such as panic effects [10], have been recently studied.

On the other hand, in recent years much attention has been paid to another type of traffic dynamics, taking place in heterogeneous media: the Internet traffic. Similar insights have been considered: free/congested phase transitions have been observed, explaining the real behavior of the Internet performance (for instance, [3] and subsequent works). Other complexity features such as information transfer in the transition neighborhood [11] or self-organized criticality [8] have been also addressed in these systems. Some evident analogies between highway and Internet systems have even been put forward [12]. The main difference between both hallmarks is that in the case of Internet traffic models, the topology of the underlying network of interactions [13] is inevitably coupled to the internal traffic dynamics, in such a way that the emergent behavior that takes place is related to the interplay between complex collective dynamics and complex interaction topologies. As a matter of fact, it is now well known that complex interaction topologies strongly affect the dynamics (for instance, percolation and epidemic thresholds have been found to be highly topology-dependent [15,16,17,18,19]), so that proper modelling of such phenomena should take good care of this fact.

Here we will handle a system quite similar to the former ones: the Air Trans-

portation System (ATS). This one is formed by a spatially extended network that physically covers a wide range of the world, made up by weighted nodes (airports of different characteristics) and links (flight routes) through which aircraft flows diffuse. As far as the latter system shows striking similarities with the highway system or Internet, in terms of non-linear coupling of local dynamics, queueing generation and congestion propagation phenomena, it is straightforward that the ATS merits a complex system insight. Some recent studies have focused on the air transportation network topology [20] and its structure [21,22] or on its application to real epidemic spreading [23,24]. Much on the contrary, the air navigation modelling state-of-art focusses on local models and uncoupled network models (for instance, [25,26,27,28]), rather than global models that take into account the nonlinear coupling effects. While some traffic dynamics systems in complex topologies have been recently performed [29,30,14], specific systems that address the Air traffic modelling are somehow lacking. In this paper we present a network based model of the ATS that simulates the effect of traffic dynamics. In section II we present the model, in terms of the network definition and the local dynamical rules. A random network topology is chosen in a baseline approach, in order to study the effect of local dynamical rules in the global behavior without any further source of complexity. In section III we point out the emergence of a jamming transition in the dynamics of aircraft diffusion, which distinguishes a regime where the average amount of queues in the network is null (efficient regime) from a regime where this average value is non null due to bottleneck generation (inefficient regime). Moreover, we show that the transition is critical. In section IV we extend the neutral model by embedding the system in heterogeneous networks. To this end we generate the (real) European air transportation network, formed by 858 European airports and 11170 flight routes connecting them. We firstly show that this network exhibits a scale-free topology, in good agreement with previous results for the worldwide transportation network [20] (indeed we show that similar exponents appear). The simulations suggest that the dynamics in this real network are qualitatively similar to what is found for the random (neutral) topology, what means that the neutral model is -at least for networks of comparable sizes- robust against changes in the network topology. In section V we provide some conclusions and depict some further work.

2 The model

We will model the ATS as a complex directed network where some dynamics take place. In the network, each node is an airport, and two nodes are linked if there exist a flight route between them (note that between two linked nodes, both directions are defined). Each node is weighted in order to characterize the

airport's design capacity (the design capacity stands for the maximal number of aircrafts per time unit that an airport can handle in an ideal situation). Each link will also be weighted in order to implement a metric layer in the network: each link weights characterizes the Euclidean distance between node pairs. Instead of the adjacency matrix alone, which fully characterizes an undirected graph [31], this co-weighted directed network will be characterized by a triple, formed by an adjacency matrix (that describes the topology structure), a distance matrix (which describes the geometric structure and the link weights) and a design capacity vector (which describes the node weights). As a neutral model, we have chosen a random network topology [31]. Observe at this point that while further relaxation of this neutrality will be performed in the section IV (where we run the model in a realistic non Poissonian network), it is necessary in a baseline approach to run the model in a network whose topological complexity may not have an effect in the global dynamics. This random network has $n = 100$ nodes where:

- (i) The node's weights (design capacities) are chosen randomly from a uniform distribution $U[1, 1000]$ and are fixed for different network geometries.
- (ii) The nodes have been linked randomly, with a link probability $p = 0.2$ (the mean degree is consequently $\langle k \rangle = p(n - 1)/2 \simeq 10$).
- (iii) The link's weights are integers chosen randomly from a uniform distribution $U[1, 10]$, characterizing the number of time steps that a given flow will need to cover the distance between two given nodes.

In addition, the real capacity RC of each node will be updated each time step as a percentage of its design capacity DC , modelled stochastically in the following way:

$$RC(t) = DC(1 - \xi), \quad (1)$$

where ξ is a random variable extracted from a uniform distribution $U[0, z]$, z characterizing the noise level. Note that this modelling is quite intuitive: the performance of individual airports deviates from their design values due to a large number of variables (changing weather conditions, runways availability, unexpected failure of software/hardware infrastructure, human related effects, to cite but a few) that can be simulated by a fitted random variable [32]. Therefore, when z is small, we assume that the node performance indicators are quite reliable and that unexpected events are not likely to occur. A large value of z denotes, on the contrary, a node restricted to high fluctuations in its performance indicators.

An initial number of aircrafts (aircraft density) is distributed uniformly over the network. Then these flows diffuse between the nodes along the links. When flows travelling across different links reach a given node i , the local rules which

apply are the following:

- (i) The input flow IF of the node i is a sum of the corresponding flows coming from i 's nearest neighbors that reach i in that step (note that distance effects are relevant at this point).
- (ii) There is a balance between the input flow and the queue generation in the following terms: if the input flow exceeds the real capacity, an amount equal to the real capacity will constitute the output flow (OF) and the difference will remain as a queue $IF > RC \rightarrow OF = RC$. The output flow is then distributed in the out-links proportionally to the weights of the corresponding target nodes.
- (iii) If the input flow is smaller than the node's real capacity the output flow is then formed by the input flow plus any remaining queue flow Q (until reaching the real capacity), $IF < RC \rightarrow OF = IF + Q$.
- (iv) When the queue of any nearest neighbor's node exceeds a given threshold Q_{t1} , the real capacity RC of node i reduces to zero in that step provided that its own queue does not exceed a given value Q_{t2} (network coupling rule). This rule mimics the following effect: a flight departure connecting two airports will be delayed if the target airport is congested (Q_{t1}). However if the departure airport is congested too, this aircraft will need to get out anyways (Q_{t2}).

Note that the updating is parallel and that as long as there is a metric layer defined in the network, the nodes updating is not synchronized.

3 Phase transition

As the flow diffuses along the network, the interplay between the network's topology, the distance desynchronization effects, the capacities stochastic updating and the coupling local rules lead to highly nonlinear dynamics. In order to study the global behavior of the network, we may define $Q_i(t)$ as the evolving queue amount at node i and P as the percentage of aircrafts that are not stuck in a node's queue, measured in the steady state. Note that P actually measures the network's efficiency as far as it gives the flow rate which is diffusing as compared to the flow rate which is stuck. We have run several Monte Carlo simulations of the diffusion of a number of aircraft (the so called aircraft density) all over the network and have measured the queue generation pattern. In figure 1 we have represented the steady values of P , as a function of the total aircraft density (note that results have been averaged over 500 network realizations). For these concrete simulations we have set $z = 0.1$, $Q_{t1} = 4000$, and $Q_{t2} = 4000$. Note that two separated regimes arise, the first where every aircraft is constantly diffusing (null amount of queues and conse-

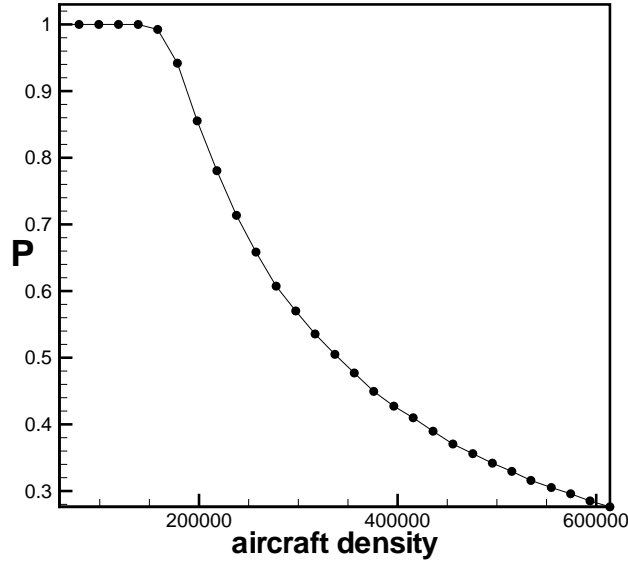


Fig. 1. Phase diagram relating the percentage of diffusing aircraft P as a function of the network aircraft density (each dot is the average of 500 network realizations). Two separated regimes arise, namely: a free (laminar) phase for low densities which stands for the efficient regime, where every aircraft is constantly diffusing along the network without any queue constraint ($P = 1$), and a congested (turbulent) phase for high densities where bottlenecks arise (inefficient regime) and the aircraft diffusivity sharply decreases, giving rise to a jamming transition.

quently $P = 1$) and the second where the amount of queues is non-null and the percentage of diffusing aircraft decreases, giving rise to a sort of jamming transition. We will quote the free phase as the efficient phase, as far as a null amount of queues denotes an efficient network. On the opposite, the congested phase will be quoted as the inefficient phase. As far as P characterizes the rate of aircraft which are diffusing, we can deduce that there is a threshold that distinguishes free moving behavior from congestion propagation. Figure 1 is indeed related to the so-called fundamental diagram in the traffic dynamics literature [1]. Such pattern of free/congested behavior have already been described in both highway traffic and in computer network traffic models, as commented in the introduction section. In the air transport hallmark, this behavior can be interpreted in the following terms: if the public demand (or eventually the slot adjudication) exceeds a given threshold in terms of number of demanded flights/slots per day, the network will decrease its performance in terms of delay rates increase in a prominent way. This kind of studies may thus constitute a key indicator of the adequate balance between public flight demand and capacity supply.

In order to provide a deeper insight of the dynamical process taking place in

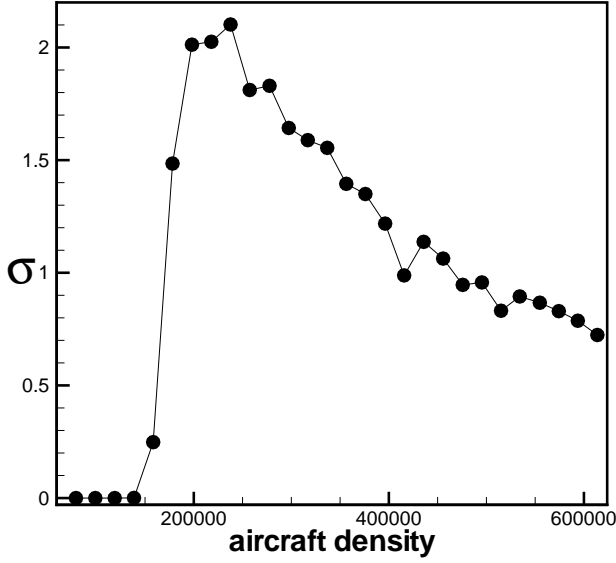


Fig. 2. Variance of P , measuring the system's fluctuations, for the same simulations as figure 1 (results have been averaged over 500 network realizations). Note that σ reaches a maximum in a neighborhood of the transition, what is characteristic of a critical phenomenon (any shift is due to finite size effects). System's uncertainty becomes maximal in this region.

the system, in figure 2 we have plotted the variance of P , σ , as a function of the aircraft density, for the same simulations as for figure 1. This variable characterizes the fluctuations that the system has, that is to say, the sensibility of the system regards to small perturbations, i.e. its uncertainty [11]. Note that the variance reaches a peaked maximum in a neighborhood of the jamming transition (any shift is due to finite size effects): the system reaches the maximal degree of unpredictability close to the transition point. This critical behavior is in turn suggesting that some information measures -such as the information entropy- may reach a maximum in this region.

We may also shed light into the microscopic dynamical evolution of the system: in figure 3 we have plotted the amount of queues in every node as a function of time $Q_i(t)$, for three different aircraft density conditions. In the upper part of the figure, the network aircraft density has been set such that the system belongs to the free phase (according to figure 1). Note that $Q_i(t)$ quickly evolves to a null value (the network is able to absorb the transient queues and the system quickly evolves into a free moving regime): in this situation the aircraft flows are constantly diffusing over the network without any queue constraint. In the bottom part of the figure, the network is in the congested phase. We can see how after a transient where many node's queues converge to a null or confined value, different nodes convert in bottlenecks, such that its queues increase monotonically in time. In this situation, a high amount of aircrafts are stuck in the bottlenecks queues, and as a consequence

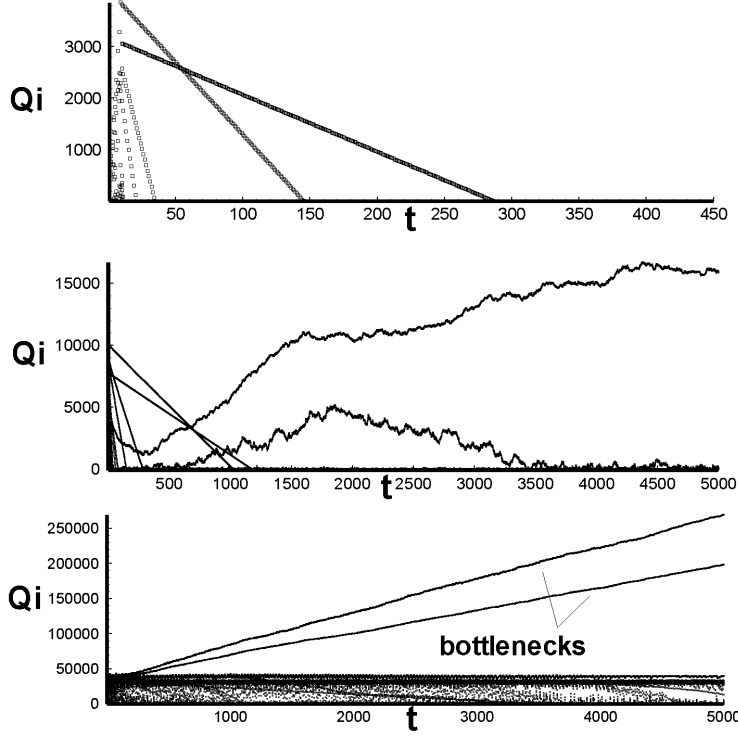


Fig. 3. Temporal evolution of the node's queues, for three different initial aircraft densities: (up) for low aircraft densities, the system quickly evolves into a free phase with no presence of queues, (middle) for medium aircraft densities, almost every node's queue converges to a null value except for some nodes where the queues evolution mimic a brownian-like motion, (down) for high aircraft densities, the system is quickly congested due to the birth of several bottlenecks.

the flow diffusion is poor. In the middle part of the figure, the network regime is in a neighborhood of the critical point (according to figure 1). We can see how the majority of the node's queues still converge to a null value, except for some nodes, whose queues are large and oscillate in an erratic trend, much in the way of a fractal random walk (fractal behavior is typical of a critical state). The evolving values of $Q_i(t)$ are indeed time series. In figure 4 we have represented a spectral analysis of one of these erratic time series. We have plotted the values of the Power spectral density $S(f)$ in a log-log scale: the system clearly evidences $1/f^\beta$ noise, as long as $S(f) \sim f^{-\beta}$ with $\beta = 1.85 \pm 0.1$. This feature clearly indicates that the system is critical, in the sense that it is highly sensible to small perturbations and is consequently more unpredictable, as seen in figure 2.

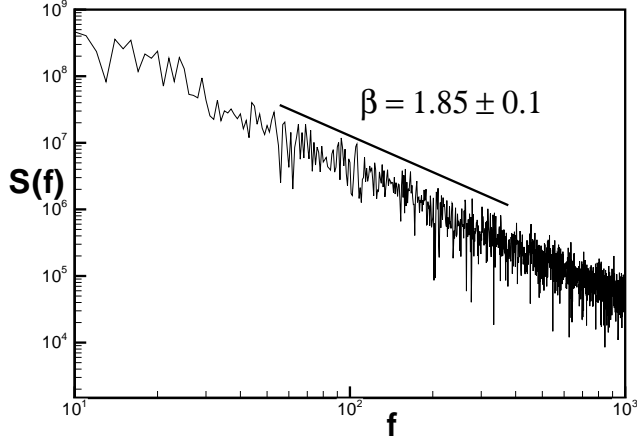


Fig. 4. Power spectral density $S(f)$ of the temporal evolution of a node's queue in the critical state. The plot is log-log: the node's queue displays fractal behavior as long as the power spectrum is of the shape $S(f) \sim 1/f^\beta$, where the best fitting provides $\beta = 1.85 \pm 0.1$.

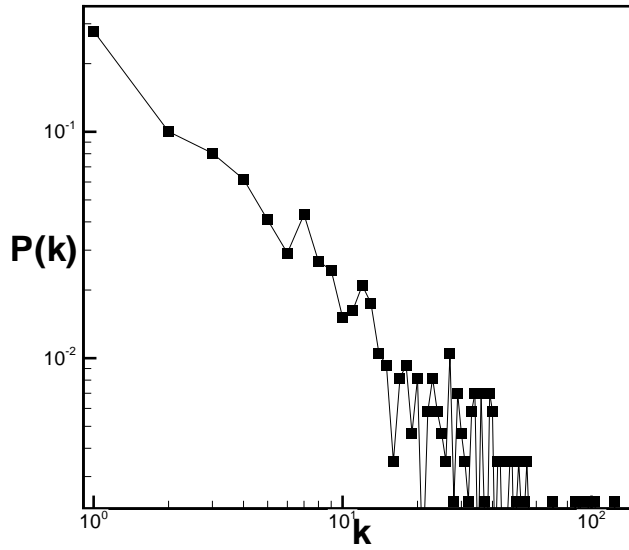


Fig. 5. Plot in log-log of the degree distribution $P(k)$ of the European air transportation network, formed by 858 European airports connected by 11170 actual flight routes. The network shows scale-free behavior $P(k) \sim k^{-\alpha}$, $\alpha = 1.1 \pm 0.1$, in good agreement with what was previously found for the worldwide air transportation network [20].

4 Real data: European air transportation network

Real transportation networks topologies differ from random Poissonian topology as their degree distribution seems to be heavy-tailed [20,21]. Since dynamics running on different topologies may eventually give rise to different macroscopic behaviors, we chose a random topology as the neutral embedding network. Here, in order to study the preceding model in more realistic scenarios, we have embedded our model in a real air transportation network. The nodes of this network represent the 858 relevant European airports (in terms of aircraft capacity and actual traffic). The adjacency matrix encodes real flight routes while the distance matrix encodes the Euclidean distance between each pair of connected airports (note that the effect of the earth's curvature has been taken into account in the distance calculations). Moreover, the airports weights (design capacities) have been estimated with real traffic data.

Prior to simulating the traffic dynamics, we have calculated some topological features of this European network. Concretely, in figure 5 we have plotted in log-log the degree distribution $P(k)$ of this network. A power law relation $P(k) \sim k^{-\alpha}$ holds: the European transportation network seems to be scale-free, in good agreement with topological signatures of the worldwide air transportation network [20] (note that as the network is not too large -858 nodes-, statistics evidence some finite size effects as noisy tails) and the Indian network [22]. The slope $\alpha = 1.1 \pm 0.1$ which is, interestingly, in good agreement with the one previously found for the worldwide air transportation network [20] but different from the one for the Indian case [22]. This fact may eventually suggest that similar mechanisms hold for the growing of worldwide and European networks. Furthermore, note that in the neutral model, the design capacity of each airport was a random variable extracted from a uniform distribution. However, in the real network the nodes weight distribution (that is, the distribution of design capacities) is no longer uniform but heavy-tailed, since the capacity of an airport is correlated with the number of flight routes that it possesses, that is, its degree (the values have been extracted from traffic data providing the European airport's daily activity).

Then, we have simulated the diffusion of a given number of aircraft through this network according to our traffic model, for values of $Q_{t1} = 4000$, $Q_{t2} = 4000$ and $z = 0.1$ (the same values as for the neutral model). The steady values of the network's efficiency P versus the total number of aircraft is plotted in figure 6. Note that the results are qualitatively similar as those found for the random (neutral) topology: above a certain threshold, the aircraft diffusion sharply decreases due to congestion effects. Quantitative differences between both scenarios are however not easy to point out.

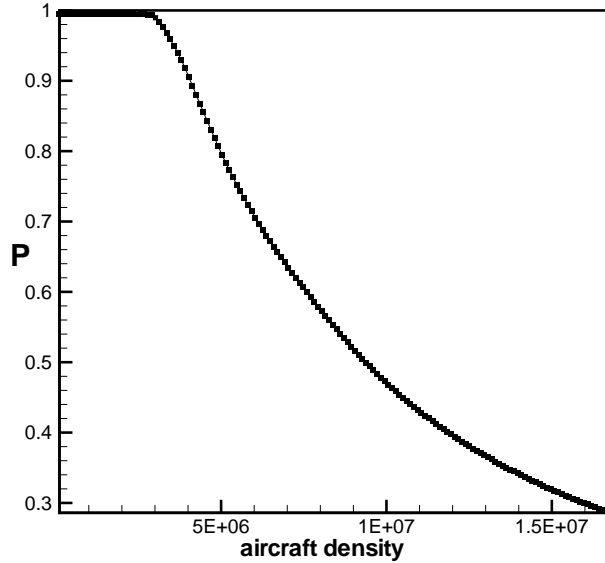


Fig. 6. Phase diagram relating the percentage of diffusing aircraft P as a function of the network aircraft density, as result of the Monte Carlo simulation of the traffic model embedded in the European air transportation network (scale-free network of formed by 858 nodes). Results are qualitatively similar to those found within a random network.

5 Summary

In this work we have introduced a network based model which simulates the Air Transportation traffic dynamics. Coupled to the network definition, local rules modelling the airport's behavior have been defined in the nodes. As a baseline study, a random network topology with a metric layer has been chosen. A critical transition distinguishing a free diffusing aircraft regime from a congested one has been found numerically. This behavior is robust against changes in the network topology, since simulations running in real air transportation networks (which we have found are scale-free) are qualitatively similar to those obtained in random (neutral) topologies. Further work should refine this baseline traffic model, in terms of more realistic rules and scenarios, systematic analysis of CDM rules, different noise implementations, and quantitative analysis of delay patterns. Note that the time restrictions that a schedule introduce in the network's availability may also have an important effect on the system dynamics [33], an issue that will also be investigated in the future.

In any case, this kind of methodologies focusing on complex cooperative behavior may be a starting point for the development of strategies and decision making procedures in order to optimize the dynamics of the Air Transportation System, as well as other network-based transportation systems.

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References

- [1] Helbing D., Rev. Mod. Phys. **73** (2001) pp. 1067-1141.
- [2] Chowdhury D. and Santen L., Phys. Rep. **329**, 199 (2000).
- [3] Ohira T. and Sawatari R., Phys. Rev. E **58**, 193 (1998).
- [4] Cuesta J.A., Martinez F.C., Molera J.M., and Sanchez A. Phys. Rev. E **48**, 4175 (1993) (R).
- [5] Krauss S., Wagner P., and Gawron C. Phys. Rev. E **55**, 5597 (1997).
- [6] Eisenblatter B., Santen L., Schadschneider A., and Schreckenberg M., Phys. Rev. E **57**, 1309 (1998).
- [7] Lubeck S., Schreckenberg M., and Usadel K.D., Phys. Rev. E **57**, 1171 (1998).
- [8] Valverde S. and Sole R.V. Physica A **312** (2002) 636-648.
- [9] Nagel K. and Schreckenberg M. J.Phys I France **2** (1992) 2221-2229.
- [10] Helbing D., Farkas I.J. and Vicsek T. Nature **407** (2000).
- [11] Sole R.V. and Valverde S. Physica A **289** (2001) 595-605.
- [12] Gabor S. and Csabai I., Physica A **307** (2002) 516-526.
- [13] Albert R. and Barabasi A.L., Rev. Mod. Phys **74** (2002) 47-97.
- [14] Echenique P., Gomez-Gardeñes J., and Moreno Y. Europhys. Lett. **71**, (2005) 325-331.
- [15] Callaway D.S., Newman M.E.J., and Watts D.J. Phys. Rev. Lett. **85**, 5468 (2000).
- [16] Cohen R., Erez K., Ben Avraham D., and Havlin S. Phys. Rev. Lett. **86**, 3682 (2001).
- [17] Pastor-Satorras R. and Vespignani A. Phys. Rev. Lett. **86**, 3200 (2001).
- [18] Pastor-Satorras R. and Vespignani A. Phys. Rev. E **63**, 066117 (2001).
- [19] Pastor-Satorras R. and Vespignani A. Handbook of Graphs and Networks: From the Genome to the Internet (Wiley-VCH, Berlin, 2002)

- [20] Guimera R., Mossa S., Turtleschi A., and Amaral L.A.N., Proc. Natl. Acad. Sci. USA **102**, 22 (2005).
- [21] Guimera R. and Amaral L.A.N., Eur. Phys. J. B **38**, 2 (2004).
- [22] R. Bagler, Physica A **387**, 12 (2008).
- [23] Collizza V., Barrat A., Barthelemy M., and Vespignani A., Proc. Natl. Acad. Sci. USA **103**, 7 (2006).
- [24] Collizza V., Barrat A., Barthelemy M., Valleron A.J., and Vespignani A., PLOS Medicine **4**, 1 (2007).
- [25] Donohue G.L., Journal of Air Traffic Control (1999).
- [26] Sanso B. and Soumis F. IEEE Transactions on Reliability **40** 1991 1.
- [27] Barnhardt C. and Schneur R.R., Operations Research **44**, 6 (1996).
- [28] Long D., Lee D., Johnson J., Gaier E. and Kostiuk P., NASA/CR1999-208988.
- [29] Hu M.-B., Wang W.-X., Jiang R., Wu Q.-S. and Wu Y.-H., Phys. Rev. E **75**, 036102 (2007).
- [30] Zhao L., Cupertino T.H., Park K. and Lai Y.-C., Chaos **17**, 043103 (2007).
- [31] Bollobas B., Modern Graph Theory, Springer, Berlin (2002)
- [32] Gardiner C.W., Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences Springer, Berlin, 2004
- [33] Zanin M., Lacasa L. and Cea M., *Chaos* (2009, in press).